QUICK REFERENCE EQUATIONS

MATHEMATICAL FORMULAS

Pythagorean theorem

$$A^2 + B^2 = C^2 \quad \textbf{(1.5)}$$

Trigonometric functions

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} \quad \textbf{(1.6)}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \quad \textbf{(1.7)}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} \quad \textbf{(1.8)}$$

$$\theta = \arcsin\left(\frac{\text{opposite side}}{\text{hypotenuse}}\right) \quad \textbf{(1.9)}$$

$$\theta = \arccos\left(\frac{\text{adjacent side}}{\text{hypotenuse}}\right) \quad \textbf{(1.10)}$$

$$\theta = \arctan\left(\frac{\text{opposite side}}{\text{adjacent side}}\right) \quad \textbf{(1.11)}$$

LINEAR KINEMATICS

Average speed

$$\bar{s} = \frac{l}{\Delta t} \quad \textbf{(2.5)}$$

Average velocity

$$\bar{v} = \frac{d}{\Delta t} \quad \textbf{(2.6)}$$

Average acceleration

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{V_f - V_i}{\Delta t} \quad \textbf{(2.9)}$$

PROJECTILE EQUATIONS

Vertical motion (y)

Vertical position:

$$y_f = y_i + v_i\,\Delta t + \frac{1}{2}g(\Delta t)^2 \quad \textbf{(2.14)}$$

$$y_f = \frac{1}{2}g(\Delta t)^2 \quad \text{if } y_i = 0 \text{ and } v_i = 0 \quad \textbf{(2.16)}$$

Vertical velocity:

$$v_f = v_i + g\,\Delta t \quad \textbf{(2.11)}$$

$$v_f^2 = v_i^2 + 2g\,\Delta y \quad \textbf{(2.15)}$$

$$v_{peak} = 0 \quad \textbf{(2.19)}$$

$$y_f = g\,\Delta t \quad \text{if } y_i = 0 \text{ and } v_i = 0 \quad \textbf{(2.17)}$$

$$v_f^2 = 2g\,\Delta y \quad \text{if } v_i = 0 \quad \textbf{(2.18)}$$

Vertical acceleration:

$$a = g = -9.81 \text{ m/s/s} \quad \textbf{(2.10)}$$

Horizontal motion (x)

Horizontal position:

$$x = v\,\Delta t \quad \textbf{(2.26)}$$

Horizontal velocity:

$$v = v_f = v_i = \text{constant} \quad \textbf{(2.22)}$$

Horizontal acceleration:

$$a = 0 \quad \textbf{(2.23)}$$

Other equations governing projectile motion

Time of flight:

$$\Delta t_{up} = \Delta t_{down} \quad \text{if } y_f = y_i \quad \textbf{(2.20)}$$

$$\Delta t_{flight} = 2\,\Delta t_{up} \quad \text{if } y_f = y_i \quad \textbf{(2.21)}$$

Parabolic equation:

$$y_f = y_i + v_{y_i}\left(\frac{x}{v_x}\right) + \frac{1}{2}g\left(\frac{x}{v_x}\right)^2 \quad \textbf{(2.27)}$$

LINEAR KINETICS

Weight

$$W = mg \quad \textbf{(1.2)}$$

Static and dynamic friction

$$F_s = \mu_s R \quad \textbf{(1.3)}$$

$$F_d = \mu_d R \quad \textbf{(1.4)}$$

Static equilibrium

$$\sum F = 0 \quad \textbf{(1.12)}$$

$$\sum F_x = 0 \quad \textbf{(1.13)}$$

$$\sum F_y = 0 \quad \textbf{(1.14)}$$

Newton's 1st Law – Law of inertia

$$v = \text{constant} \quad \text{if } \sum F = 0 \quad \textbf{(3.1a)}$$

or

$$\sum F = 0 \quad \text{if } v = \text{constant} \quad \textbf{(3.1b)}$$

Linear momentum

$$L = mv \quad \textbf{(3.6)}$$

Conservation of momentum

$$L = \text{constant} \quad \text{if } \sum F = 0 \quad \textbf{(3.7)}$$

$$L_x = \text{constant} \quad \text{if } \sum F_x = 0 \quad \textbf{(3.8)}$$

$$L_y = \text{constant} \quad \text{if } \sum F_y = 0 \quad \textbf{(3.9)}$$

$$L_i = \sum(mu) = m_1u_1 + m_2u_2 + m_3u_3 + \ldots = m_1v_1 + m_2v_2 + m_3v_3 + \ldots = \sum(mv) = L_f = \text{constant} \quad \text{if } \sum F = 0 \quad \textbf{(3.11)}$$

Perfectly elastic collision of two objects

$$m_A u_A = m_B v_B \quad \textbf{(3.14)}$$

Perfectly inelastic collision of two objects

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v \quad \textbf{(3.15)}$$

Coefficient of restitution

$$e = \left|\frac{v_1 - v_2}{u_1 - u_2}\right| = \left|\frac{v_2 - v_1}{u_1 - u_2}\right| \quad \textbf{(3.16)}$$

Newton's 2nd Law – Law of acceleration

$$\sum F = ma \quad \textbf{(3.18)}$$

$$\sum F_x = ma_x \quad \textbf{(3.19)}$$

$$\sum F_y = ma_y \quad \textbf{(3.20)}$$

Impulse–momentum equation

$$\sum \bar{F}\,\Delta t = m\,(v_f - v_i) \quad \textbf{(3.26)}$$

Universal law of gravitation – gravitational force

$$F = G\left(\frac{m_1m_2}{r^2}\right) \quad \textbf{(3.27)}$$

WORK, POWER, AND ENERGY

Work

$$U = \bar{F}(d) \quad \textbf{(4.2)}$$

Kinetic energy

$$KE = \frac{1}{2}mv^2 \quad \textbf{(4.4)}$$

Gravitational potential energy

$$PE = Wh \quad \textbf{(4.5)}$$

Strain energy

$$SE = \frac{1}{2}k\,\Delta x^2 \quad \textbf{(4.7)}$$

Work–energy principle

$$U = \Delta E \quad \textbf{(4.8)}$$

Power

$$P = \frac{U}{\Delta t} \quad \textbf{(4.12)}$$

$$P = \bar{F}\bar{v} \quad \textbf{(4.13)}$$

ANGULAR KINEMATICS

Angular position measured in radians

$$\theta = \frac{\text{arc length}}{r} = \frac{l}{r} \quad \textbf{(6.1)}$$

Angular displacement and arclength

$$l = \Delta\theta r \quad \textbf{(6.4)}$$

Average angular velocity

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{\Delta t} \quad \textbf{(6.6)}$$

Angular velocity and linear velocity

$$v_T = \omega r \quad \textbf{(6.8)}$$

Average angular acceleration

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_f}{\Delta t} \quad \textbf{(6.9)}$$

Tangential acceleration

$$a_T = \alpha r \quad \textbf{(6.10)}$$

Centripetal acceleration

$$\alpha_r = \frac{v_T^2}{r} \quad \textbf{(6.11)}$$

$$\alpha_r = \omega^2 r \quad \textbf{(6.12)}$$

ANGULAR KINETICS

Torque

$$T = F \times r \quad \textbf{(5.1)}$$

Static equilibrium

$$\sum T = 0 \quad \textbf{(5.2)}$$

Center of gravity

$$\sum (W \times r) = (\sum W) \times r_{cg} \quad \textbf{(5.3)}$$

Moment of inertia

$$I_a = \sum m_i r_i^2 \quad \textbf{(7.1)}$$

$$I_a = mk_a^2 \quad \textbf{(7.2)}$$

Moment of inertia – parallel axis theorem

$$I_b = I_{cg} + mr^2 \quad \textbf{(7.3)}$$

Angular momentum

$$H_a = I_a \omega_a \quad \textbf{(7.6)}$$

Angular momentum of the human body

$$H_a = \sum (I_i \omega_i + m_i r_{i/cg}^2 \omega_{i/cg}) \quad \textbf{(7.7)}$$

Conservation of angular momentum

$$H_i = I_i \omega_i = I_f \omega_f = H_f = \text{constant} \quad \text{if } \sum T = 0 \quad \textbf{(7.9)}$$

Angular version of Newton's 2nd law

$$\sum T_a = I_a \alpha_a \quad \textbf{(7.12)}$$

$$\sum \overline{T_a} = \frac{\Delta H_a}{\Delta t} = \frac{(H_f - H_i)}{\Delta t} \quad \textbf{(7.13)}$$

Angular impulse-momentum

$$\sum \overline{T_a} \Delta t = (H_f - H_i)_a \quad \textbf{(7.15)}$$

FLUID MECHANICS

Pressure

$$P = \frac{F}{A}$$

Density

$$\rho = \frac{m}{V} \quad \textbf{(8.3)}$$

Drag force

$$F_D = \frac{1}{2} C_D \rho A v^2 \quad \textbf{(8.5)}$$

Lift force

$$F_L = \frac{1}{2} C_L \rho A v^2 \quad \textbf{(8.6)}$$

MECHANICS OF MATERIALS

Stress

$$\sigma = \frac{F}{A} \quad \textbf{(9.1)}$$

Shear stress

$$\tau = \frac{F}{A} \quad \textbf{(9.2)}$$

Strain

$$\varepsilon = \frac{l - l_0}{l_0} \quad \textbf{(9.4)}$$

Elastic modulus

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} \quad \textbf{(9.5)}$$

ABBREVIATIONS FOR VARIABLES AND SUBSCRIPTS USED IN EQUATIONS

Variables

a = instantaneous linear acceleration
$\bar{a}$ = average linear acceleration
A = area
C_D = coefficient of drag
C_L = coefficient of lift
d = displacement
e = coefficient of restitution
E = energy
E = elastic modulus or Young's modulus
F = force
$\bar{F}$ = average force
F_d = dynamic friction force
F_s = static friction force
ΣF = net force = sum of forces
g = acceleration due to gravity
G = gravitational constant
h = height
H = angular momentum
I = moment of inertia
k = radius of gyration
k = stiffness or spring constant
KE = kinetic energy
l = distance traveled or length
L = linear momentum
m = mass
P = power
P = pressure
P = force
PE = gravitational potential energy
r = radius
r = moment arm
R = normal contact force
s = instantaneous linear speed
$\bar{s}$ = average linear speed
t = time
T = torque
u = pre-impact velocity
U = work done
v = instantaneous linear velocity
v = post-impact velocity
$\bar{v}$ = average linear velocity
V = volume
W = weight
x = horizontal position
y = vertical position
α = instantaneous angular acceleration
$\bar{\alpha}$ = average angular acceleration
Δ = change in … = final – initial
ε = strain
μ = coefficient of friction
ρ = density
σ = stress
Σ = sum of …
τ = shear stress
θ = angular position
ω = instantaneous angular velocity
$\bar{\omega}$ = average angular velocity

Subscripts

d = dynamic
cg = center of gravity
D = drag
f = final or ending
i = initial or starting
i = one of a number of parts
L = lift
o = original or undeformed
r = radial
s = static
T = tangential
x = horizontal
y = vertical